Solow Model

Endogenous Growth

Topic 5: Long-Run Economic Growth

Lei Pan

School of Arts Australian Catholic University

Chapter Outline

- Discuss the sources of economic growth and the fundamentals of growth accounting.
- Explain the factors affecting long-run living standards in the Solow model.
- Endogenous Growth Theory.
- Discuss government policies for raising long-run living standards.

Introduction

- Countries have grown at very different rates over long spans of time.
- Why should we care about growth?
 - Output grows faster than population -> GDP per capita \uparrow -> our measure of the average standard of living \uparrow
- Big questions on long-run economic growth:
 - What determines long-run economic growth?
 - Why some countries are rich while some are poor?
- Explaining two striking facts
 - Why are there large cross-country differences in GDP per capita?
 - Why countries have grown at very different rates of long spans of time?

Sources of Growth

Solow Model

Endogenous Growth

Economic Growth in Eight Major Countries, 1870-2008

	Levels of real GDP per capita				
Country	1870	1913	1950	2008	Annual growth rate 1870–2008
Australia	3.273	5,157	7.412	25.301	1.5%
Canada	1,695	4,447	7,291	25,267	2.0
France	1,876	3,485	5,186	22,223	1.8
Germany	1,839	3,648	3,881	20,801	1.8
Japan	737	1,387	1,921	22,816	2.5
Sweden	1,359	3,073	6,769	24,409	2.1
United Kingdom	3,190	4,921	6,939	23,742	1.5
United States	2,445	5,301	9,561	31,178	1.9

Note: Figures are in U.S. dollars at 1990 prices, adjusted for differences in the purchasing power of the various national currencies.

Source: Data from Angus Maddison, *Statistics on World Population, GDP, and Per Capita GDP, 1–2008 AD* (February 2010, vertical file, copyright Angus Maddison), available at *www.ggdc.net/maddison*.

Sources of Growth

Solow Model

Endogenous Growth

The Sources of Economic Growth

• The production function:

$$Y = AF(K, N),$$

where F tells us how much output is produced for given quantities of capital and labour. The production function depends on *the state of technology*, A. The higher the state of technology, the higher output Y for a given K and a given N.

• Decompose into growth rate form (*the growth accounting equation*): $\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha_K \frac{\Delta K}{K} + \alpha_N \frac{\Delta N}{N}$

where α terms are the elasticities of output with respect to the inputs (capital and labour).

- Interpretation: An increase of 10% in A raises output by 10%. An increase of 10% in K raises output by α_K times 10%. An increase of 10% in N raises output by α_N times 10%.
- Both α_K and α_N are less than 1 due to diminishing marginal productivity.

Growth accounting

- Four steps in breaking output growth into its causes (productivity growth, capital input growth, labour input growth):
- Get data on $\frac{\Delta Y}{Y}$, $\frac{\Delta K}{K}$, and $\frac{\Delta N}{N}$, adjusting for quality changes.
- **2** Estimate α_K and α_N from historical data.
- Calculate the contributions of K and N as $\alpha_K \frac{\Delta K}{K}$ and $\alpha_N \frac{\Delta N}{N}$, respectively.
- Galculate productivity growth as the residual:

$$\frac{\Delta A}{A} = \frac{\Delta Y}{Y} - \alpha_K \frac{\Delta K}{K} - \alpha_N \frac{\Delta N}{N}$$

Sources of Growth

Solow Model

Endogenous Growth

The Steps of Growth Accounting: A Numerical Example

Step 1. Obtain measures of output growth, capital growth, and labor growth over the period to be studied.

Example: output growth = $\frac{\Delta Y}{Y} = 40\%$;

capital growth
$$=\frac{\Delta K}{K}=20\%$$

labor growth
$$=\frac{\Delta N}{N}=$$
 30%.

Step 2. Using historical data, obtain estimates of the elasticities of output with respect to capital and labor, a_K and a_{N^*}

Example:
$$a_K = 0.3$$
 and $a_N = 0.7$.

Step 3. Find the contributions to growth of capital and labor.

Example: contribution to output growth of growth in capital = $a_K \frac{\Delta K}{K} = 0.3 \times 20\% = 6\%$;

contribution to output growth of growth in labor =
$$a_N \frac{\Delta N}{N} = 0.7 \times 30\% = 21\%$$
.

Step 4. Find productivity growth as the residual (the part of output growth not explained by capital or labor).

Example: productivity growth =
$$\frac{\Delta A}{A} = \frac{\Delta Y}{Y} - \partial_K \frac{\Delta K}{K} - \partial_N \frac{\Delta N}{N}$$

= 40% - 6% - 21% = 13%.

Growth accounting and the productivity slowdown

- Denison's results for 1929-1982:
 - Entire period output growth 2.92%; due to labour 1.34%; due to capita 0.56%; due to productivity 1.02%.
 - Pre-1948 capital growth was much slower than post-1948.
 - Post-1973 labour growth slightly slower than pre-1973.
- Productivity growth is major difference
 - Entire period: 1.02%. 1929-1948: 1.01%. 1948-1973: 1.53%. 1973-1982: -0.27%.
- Productivity growth slowdown occurred in all major developed countries.

Solow Model

Endogenous Growth

Application: the post-1973 slowdown in productivity growth

- What caused the decline in productivity?
 - Measurement inadequate accounting for quality improvements.
 - The legal and human environment regulations for pollution control and worker safety, crime, and declines in educational quality.
 - Oil prices huge increase in oil prices reduced productivity of capital and labour, especially in basic industries.
 - New industrial revolution learning process for information technology from 1973 to 1990 meant slower growth.

Sources of Growth

Solow Model

Endogenous Growth

Sources of Economic Growth in the United States (Denison) (Percent per Year)

	(1) 1929–1948	(2) 1948–1973	(3) 1973–1982	(4) 1929–1982	(5) 1982–2011
Source of Growth					
Labor growth	1.42	1.40	1.13	1.34	0.99
Capital growth	0.11	0.77	0.69	0.56	1.18
Total input growth	1.53	2.17	1.82	1.90	2.17
Productivity growth	1.01	1.53	-0.27	1.02	1.06
Total output growth	2.54	3.70	1.55	2.92	3.23

Sources: Columns (1)–(4) from Edward F. Denison, *Trends in American Economic Growth*, 1929–1982, Washington, D.C.: The Brookings Institution, 1985, Table 8.1, p. 111. Column (5) from Bureau of Labor Statistics Web site, Multifactor Productivity Trends, Table XG, available at *ftp://ftp.bls.gov/pub/special.requests/opt/mp/ prod3.mfptablehis.zip*



Solow Model

Endogenous Growth

Application: the recent surge in U.S. productivity growth

- Labour productivity growth increased sharply in the second half of the 1990s.
- Labour productivity and TFP grew steadily from 1982 to 2008.

Sources of Growth

Solow Model

Endogenous Growth

Productivity Levels, 1948-2011



Productivity

- Labour productivity growth has generally exceeded TFP growth since 1995.
- Use equations to relate the differing productivity concepts:

$$\frac{\Delta Y}{Y} - \frac{\Delta N}{N} = \frac{\Delta A}{A} + \alpha_K (\frac{\Delta K}{K} - \frac{\Delta N}{N})$$

So, labour productivity growth exceeds TFP growth because of faster growth of capital relative to growth of labour.

• ICT growth (information and communications technology) may have been a prime reason.

Sources of Growth

Solow Model

Endogenous Growth

Productivity Growth, 1949-2011



Productivity

Outline

- Why did ICT growth contribute to U.S. productivity growth, but not in other countries?
 - Government regulations.
 - Lack of competitive pressure.
 - Available labour force.
 - Ability to adapt quickly.
- Why was there such a lag between investment in ICT and growth in productivity?
- Intangible capital: R&D, Firm reorganization, Worker training.

Productivity

- Similar growth in productivity experienced in past:
 - Steam power, railroads, telegraph in late 1800s.
 - Electrification of factories after WWI.
 - Transistor after WWII.
- What matters most is ability of economy to adapt to new technologies.

Two basic questions about growth

- What's the relationship between the long-run standard of living and the saving rate, population growth rate, and rate of technical progress?
- How does economic growth change over time? Will it speed up, slow down, or stabilize?

The Solow Model

- It was developed in the late 1950s by Nobel laureate Robert Solow.
- Basic assumptions and variables:
 - Size of population = Size of labour force
 - Population and work force grow at same rate n.
 - Economy is closed and G = 0:

$$C_t = Y_t - I_t$$

• Rewrite everything in per-worker terms:

$$y_t = \frac{Y_t}{N_t}; \ c_t = \frac{C_t}{N_t}; \ k_t = \frac{K_t}{N_t}$$

where k_t is also called the capital-labour ratio. The per-worker production function:

$$y_t = f(k_t)$$

Production function

• Assume no productivity growth for now (add it later), A = 1 $Y_t = F(K_t, N_t) = K_t^{\alpha_K} N_t^{1-\alpha_K}$ $\Rightarrow y_t = \frac{Y_t}{N_t} = k_t^{\alpha_K}$

Hence, the per-worker production function:

$$y_t = f(k_t)$$

Same shape as aggregate production function.

- upward sloping
- diminishing MPK

Outline

Sources of Growth

Solow Model

Endogenous Growth

The per-worker production function



Analytical method of economics

- Static analysis: There are no changes on the market (e.g. The intersection of labour supply curve and demand curve brings to labour market equilibrium.)
- **Comparative static analysis**: A comparative analysis for different statics (e.g. Labour supple curve shifts to the right which leads to a new equilibrium. Comparison between the new equilibrium with the old equilibrium.)

Note: We do not care about the **path** and **how long** it takes to reach the new equilibrium under this method.

• **Dynamic analysis**: Incorporate time dimension to the analysis. There are changes in some variables over time. It discusses the change in equilibrium as time passed by.

Steady states

- Steady state is the "equilibrium" under dynamic analysis.
 - y_t , c_t , and k_t are constant over time
- Gross investment must:
 - Replace worn out capital, dK_t .
 - Expand so the capital stock grows as the economy grows, nK_t :

$$I_t = (n+d)K_t$$

• The steady state consumption

$$C_t = Y_t - (n+d)K_t$$

• In per-worker terms, in steady state:

$$c = f(k) - (n+d)k$$

Plot of c, f(k), and (n+d)k.

Sources of Growth

Solow Model

Endogenous Growth

The relationship of consumption per worker to the capital-labour ratio in the steady state



Some Interpretations

- Increasing k will increase c up to a point.
 - This is k_G in the figure, the Golden Rule capital-labour ratio.
 - For k beyond this point, c will decline. But we assume henceforth that k is less than k_G, so c always rises as k rises.
- Suppose saving is proportional to current income:

$$S_t = sY_t$$

where s is the saving rate, which is between 0 and 1.

• Equating saving to investment

$$sY_t = (n+d)K_t$$

• The higher the output, the higher are saving and investment.

Some Interpretations

• Putting this in per-worker terms gives:

$$sf(k) = (n+d)k$$

Plot of sf(k) and (n+d)k.

- The only possible steady-state capital-labour ratio is k*. Output at that point is y* = f(k*); consumption is c* = f(k*) (n+d)k*.
- If k begins at some level other than k^* , it will move toward k^* :
 - For k below k*, saving > the amount of investment needed to keep k constant, so k rises.
 - For k above k*, saving < the amount of investment needed to keep k constant, so k falls.



Determining the capital-labour ratio in the steady state





- Take a poor country (one with low k) and a rich country (that has a high k).
- The poor country will probably be farther away from *k** than the rich country.
- Then the poor country should grow faster than the rich country and catch up.
- Given the same level of technology and human capital, same institutions, etc.
- This model says that all countries should converge to the same level.

Summary

The model implication:

- With no productivity growth, the economy reaches a steady state, with constant capital-labour ratio, output per worker, and consumption per worker.
- With no productivity growth, there is no economic growth once the economy reaches steady state.
- The fundamental determinants of long-run living standards
 - The saving rate.
 - Population growth.
 - Productivity growth.

The saving rate

- Higher saving rate (s) means higher capital-labour ratio (k*), higher output per worker (y*), and higher consumption per worker (c*).
- The saving rate has no effect on the long run growth rate of output per worker, which is equal to zero.
 - Output per worker and capital per worker are constant in the steady state.
 - If an economy wanted to increase the steady state k* every year it would have to increase savings/output every year.
- Nonetheless, the saving rate determines the level of output per worker in the long run. Other things equal, countries with a higher saving rate will achieve higher output per worker in the long run.
- Should a policy goal be to raise the saving rate?
 - Not necessarily, since the cost is lower consumption in the short run.
 - There is a trade-off between present and future consumption.

Sources of Growth

Solow Model

The effect of an increased saving rate on the steady-state capital-labour ratio



Population growth

Outline

• Higher population growth means a lower capital-labour ratio, lower output per worker, and lower consumption per worker.

Solow Model

- Should a policy goal be to reduce population growth?
 - Doing so will raise consumption per worker.
 - But it will reduce total output and consumption, affecting a nation's ability to defend itself or influence world events.
- The Solow model also assumes that the proportion of the population of working age is fixed.
 - But when population growth changes dramatically this may not be true.
 - Changes in cohort sizes may cause problems for social security systems and areas like health care.

Sources of Growth

Solow Model

Endogenous Growth

The effect of a higher population growth rate on the steady-state capital-labour ratio



Productivity growth

- The key factor in economic growth is productivity improvement.
- Productivity improvement raises output per worker for a given level of the capital-labour ratio.
- In equilibrium, productivity improvement increases the capital-labour ratio, output per worker, and consumption per worker:
 - Productivity improvement directly improves the amount that can be produced at any capital-labour ratio.
 - The increase in output per worker increases the supply of saving, causing the long-run capital-labour ratio to rise.
- Can consumption per worker grow indefinitely?
 - The saving rate can't rise forever (it peaks at 100%) and the population growth rate can't fall forever.
 - But productivity and innovation can always occur, so living standards can rise continuously.
- Summary: The rate of productivity improvement is the dominant factor determining how quickly living standards rise.

Sources of Growth

Solow Model

Endogenous Growth

An improvement in productivity



Sources of Growth

Solow Model

Endogenous Growth

The effect of a productivity improvement on the steady-state capital-labour ratio



00000000000

Outline

Application: The growth of China

- China is an economic juggernaut.
 - Population 1.4 billion people.
 - Real GDP per capita is low but growing.
 - Starting with low level of GDP, but growing rapidly.
- Fast output growth attributable to
 - Huge increase in capital investment.
 - Fast productivity growth (in part from changing to a market economy).

Solow Model

- Increased trade.
- Question: As China grows, does welfare increase?
 - Yes for higher standard of living
 - however...
 - Wei and Zhang (2011): immiserizing growth in China

Sources of Growth

Solow Model

Economic Growth in China, Japan, and the United States

Levels of real GDP per capita							
Country	1870	1913	1950	2008	Annual growth rate 1870–2008		
China Japan United States	530 737 2,445	552 1,387 5,301	448 1,921 9,561	6,725 22,816 31,178	1.9% 2.5 1.9		
Note: Figures are in U.S. dollars at 1990 prices, adjusted for differences in the purchasing power of the various national currencies. Source: Data from Angus Maddison, Statistics on World Population, GDP, and Per Capita GDP, 1–2008 AD (February 2010, vertical file, copyright Angus Maddison), available at www.ggdc.net/maddison.							

Sources of Growth

Solow Model

Endogenous Growth

Real GDP growth in China and the United States, 2001-2011



Solow Model

Endogenous Growth

Will China be able to keep growing rapidly?

- Rapid growth because of
 - use of underemployed resources.
 - using advanced technology developed elsewhere.
 - making transition from centrally-planned economy to market economy.
- Such gains may not last. So, it may take China a long time to catch up with the rest of the developed world.

Endogenous growth theory

- Solow model:
 - Keep moving productivity, k^* is growing.
 - However...
 - Productivity growth seems to be the only source of long-run growth.
- Endogenous growth theory explaining the sources of productivity growth. Aggregate production function:

$$Y = AK$$

- How do we understand constant MPK?
 - Human capital

Increase in human capital (such as increases in knowledge, skills, and training of individuals) offsets decline in *MPK* from having more physical capital.

• Research and development

Increases in physical capital generate increased technical knowledge, which offsets decline in MPK from having more physical capital.

Endogenous Growth

Implications of endogenous growth

• Suppose saving is a constant fraction of output:

$$S = sAK$$

• Since investment = net investment + depreciation:

$$I = \Delta K + dK$$

• Setting investment equal to saving implies:

$$\Delta K + dK = sAK$$
$$\frac{\Delta K}{K} = sA - d$$

• Since output is proportional to capital, $\frac{\Delta Y}{Y} = \frac{\Delta K}{K}$, so

$$\frac{\Delta Y}{Y} = sA - d$$

• What's new?

- Now saving rate affects the long-run growth rate (not true in Solow model).
- Human capital formation and R&D can affect productivity and hence, long-run growth.

Summary

- Endogenous growth theory attempts to explain, rather than assume, the economy's growth rate.
- The growth rate depends on many things, such as the saving rate, that can be affected by government policies.

Policies to affect the saving rate

- If the private market is efficient, the government shouldn't try to change the saving rate:
 - The private market's saving rate represents its trade-off of present for future consumption.
 - But if tax laws or myopia cause an inefficiently low level of saving, government policy to raise the saving rate may be justified.
- How can saving be increased?
 - One way is to raise the real interest rate to encourage saving; but the response of saving to changes in the real interest rate seems to be small.
 - Another way is to increase government saving: The government could reduce the deficit or run a surplus. But under Ricardian equivalence, tax increases to reduce the deficit won't affect national saving.

Policies to raise the rate of productivity growth

- Improving infrastructure:
 - Infrastructure: highways, bridges, utilities, dams, airports.
 - Empirical studies suggest a link between infrastructure and productivity.
 - U.S. infrastructure spending has declined in the last two decades.
- Would increased infrastructure spending increase productivity?
 - There might be reverse causation: Richer countries with higher productivity spend more on infrastructure, rather than vice versa.
 - Infrastructure investments by government may be inefficient, since politics, not economic efficiency, is often the main determinant.

Policies to raise the rate of productivity growth

- Building human capital:
 - There's a strong connection between productivity and human capital.
 - Government can encourage human capital formation through educational policies, worker training and relocation programs, and health programs.
 - Another form of human capital is entrepreneurial skill.
 - Government could help by removing barriers like red tape.

Solow Model

Endogenous Growth

Policies to raise the rate of productivity growth

- Encouraging research and development:
 - Support scientific research.
 - Fund government research facilities.
 - Provide grants to researchers.
 - Contract for particular projects.
 - Give tax incentives.
 - Provide support for science education.